Secondary School Students' Errors and Misconceptions in Understanding Algebraic Variables among Senior Secondary Schools in Sokoto State, Nigeria

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Abstract

This study investigated the students' errors and misconceptions in understanding algebraic variables among senior secondary schools in sokoto state, Nigeria. The study adopted survey research design using mixed method. Data were collected from a sample of 378 students which were selected from the total number of 32,298 SSII students in the 142 public senior secondary schools in Sokoto State. Two instruments were developed and validated by the experts. Students' Algebraic Error Test (SAET) and Algebraic Misconceptions Interview Questionnaire (AMIQ) with reliability coefficients of 0.82 and 0.86 were obtained respectively. Data collected were analyzed using descriptive statistics and thematic content analysis. Findings of this study revealed that, majority of the students had difficulties in understanding the algebraic variable and most of the errors committed by students are not ordinary errors per se rather are misconceptions. The identified categories of errors include: assigning labels, arbitrary values, verbs and constants for variables, misinterpreting of two variables, lack of understanding the concept of variable and forming wrong equations as answers. Based on these findings it was recommended that Mathematics teachers should involve their students in activities that will aid their learning algebraic variables. These will make students to be conscious of the various Algebraic variables, concepts and improve to overcome their errors and misconceptions.

Keywords: Algebraic Variable, Errors, Misconceptions, Interest

Introduction

Mathematics has been recognized as bedrock for successful functioning of all aspects of human endeavor and that no nation can achieve any measure of scientific and technological advancement without mathematics. The knowledge of mathematics is a basic pre-requisite in the quest for present day scientific and technological advancement. The idea of numbers, structures and relationships which make mathematics connotes stand out in medicine, business, education, agriculture, and every field of human endeavour (Ningsih, Lefrida & Murdiana, 2024). In the 21st century in particular, sound mathematical background is needed for life sustenance and workplace competence. Mathematics is the queen of science and a tool for scientific and technological advancement, an indispensable tool for effective use of electronic resources for national development. It is also a way to communicate ideas, and perhaps more than anything, it is a way of reasoning that is unique to human beings (Akissani, Kajuru, Falalu & Aliyu, 2025). However, Olutosin (2007) described mathematics as an instrument to ease or facilitate the learning of other subjects and that, the importance of mathematics permeates all aspects of human endeavour.

Algebra is described as the logical outgrowth of arithmetic (Egodawatte, 2011). According to Obunge & Adolphus (2024), Algebra as a branch of mathematics in which symbols (usually letters) represent unknown numbers in Mathematical Equations, Algebra allows the basic operations of arithmetic's, such as addition, subtraction, multiplication, and division, to be performed without using specific numbers. The history of Algebra began in ancient Egypt and Babylon, where people learned to solve linear and quadratic Equations as well as intermediate Equations where by several unknowns are involved. However, Algebra is extensive in concepts, forms, structure, and applications since it is studied at almost all educational levels (NCTM, 2019). Despite modifications in concept sequence, categories, complexity, and application, additionally, Algebra keeps its name throughout Secondary and Postsecondary Education. In Senior Secondary Schools, for instance, Algebra was split into Equations (Basic, Simultaneous and Quadratic), set theory, inequality, and variation, each of which needed distinct problem-solving strategies (Mary, Mihis and Ndethiu, 2016). More so, according to Adeniran and Lambaya, (2022), one type of thinking needed in mathematics is algebraic thinking, in the broadest sense, algebraic thinking consists of understandings series which is needed to interpret the world by translating information or events into the language of Mathematics in order to explain and predict the phenomena, theoretically or practically. Algebra is found in many areas of life, such as, bankers who study patterns for electronically transferred funds and computer experts use digit patterns and switch to give code for complex statements (Zakaria, Chin, & Daud, 2010).

An error is the deviation from a correct solution of a problem as contextualized in Mathematics. Errors could be found as a wrongly answer problems which have flaws in the process that generated answers (Stemele & Jina, Asvat, 2024). Mathematics teachers can directly teach the correct procedure for solving and doing Mathematics correctly. While, there are common errors that students made with performing algbraic processes, students may demonstrate errors that are individual specific, (Akissani, Kajuru, Falalu & Aliyu, 2025). Thus, Siagian, et al., (2021) attributed that most of the students understood an algebraic variable as something whose value is unknown, whilst some other students comprehend a variables as a substitute for something whose value is unknown. Among the factors that influence students' weaknesses in mathematics are from the knowledge base or basic concepts and skills from previous learning (Ling, Shahril, & An, 2016). Students who fail to master the necessary skills and ideas at any level of learning, can influence their achievement at the next level. Hence, it is vital for a teacher to make an assessment of teaching and learning in the classroom and try to identify the difficulties and errors that are often experienced by students.

Misconception is a view or opinion that is incorrect based on faulty thinking or an inadequate understanding. The word misconception refers to an incorrect idea that is based on a failure to understand a concept (Obunge & Adolphus, 2024; Pramesti & Retnawati, 2019; Tekin-Sitrava, 2017). Misconceptions in learning Mathematics is a common occurrence across different cohorts in the use of the equal sign, symbolism, letters of usage, vocabulary and the order of operation (Egodawatte, 2011). Teachers should embrace errors and misconceptions in their teaching and should not regard them as obstacles to learning but rather engage with them for better understanding of algebraic concepts by students. (Fitria & Susanto, 2023; Ndemo & Ndemo, 2018). Similarly, misconception indicates a person's wrong understanding, and it occurs repeatedly because it is resistant and can produce wrong results (Tooher & Johnson, 2020). In view of this, the present study deemed it necessary to investigate students' errors and misconceptions which need to provide clear an explanation about the state of situation in Sokoto state, for better selection of an appropriate approach to minimize the problem for better learning outcomes and improve algebraic thinking ability of the learners.

Statement of the Problem

Algebra as crucial component of Mathematics and also an efficient way of solving certain types of problems. In spite of the importance of algebra to mathematics, scientific and technological development, it is still notable that students' performance in algebra-related topics at internal and external examinations has remained considerably poor. According to the Chief Examiners' report on mathematics in Nigeria, it has been observed that the candidates showed weaknesses in algebra-related topics. As such, it is suggested that, teachers should endeavour to pay more attention to algebra-related topics (WAEC, 2023). However, students in secondary schools, particularly in senior secondary schools, have not explored the resourcefulness of algebraic concepts, as evidenced by their poor performance (Egodawatte, 2011). This tendency may be attributable to the difficulty of learning and comprehending algebraic process in senior secondary schools. Due to the above problems, the present study deemed it necessary to investigate the students' errors and misconceptions specifically in understanding algebraic variables in Sokoto State.

Theoretical Foundation

The theoretical foundation of this study includes learning disability theory and constructivist theory. Learning disabilities are disorders that affect one's ability to understand and do mathematical calculations, or direct attention, which can lead to errors and misconceptions. Experiential learning theory (ELT) as postulated by Kolb (2014) determined four phases of ELT namely concrete experience, reflective observation, abstract conceptualization and active experimentation. This four-stage learning cycle was demonstrated through (1) concrete experience, in which students gain experience by conducting an actual training needs assessment (TNA) of a specified target population through survey, (2) reflection and observation, in which the students reflect and review the results of the TNA process before deciding on the training type, objectives, as well as the content, (3) abstract conceptualization, in which the students begins to design the training module which includes the lesson plan, materials such as slides, evaluation sheets, activities and so on, based on the information gathered from the first stage, (4) active experimentation, whereby students conduct the planned training program for the targeted audience identified in the previous phase. The students are also required to evaluate the effectiveness of the training program, followed by a reflection on the results achieved as well as areas for improvement.

Objectives:

The objectives of the study is to investigate students' errors and misconceptions in understanding algebraic variables among senior secondary schools in Sokoto state, Nigeria. Specifically, the objectives of the study are to:

1. identify the categories of errors made by senior secondary school students in understanding algebraic variables in Sokoto state, Nigeria.
2. explore the misconceptions made by senior secondary school students in understanding algebraic variables in Sokoto state, Nigeria.

Research Question

The following research questions guided this study:

1. What are the categories of errors made by senior secondary school students in understanding algebraic variables in Sokoto state, Nigeria?
2. What are the misconceptions made by senior secondary school students in understanding algebraic variables in Sokoto state, Nigeria?

Methodology

This study employed a descriptive survey research design using a mixed method which is characterized by the collection and analysis of quantitative data followed by the collection and analysis of qualitative data**.** The population of this study comprised all the 32,239 students from the 142 public senior secondary schools II in Sokoto State (M.O.E., 2024). However, the sample size of 378 Students was obtained using the table for determining sample size from the given population using Research Advisor (2006). The instrument for data collection used in this study consists of two different types of instruments, quant*it*ative and qualitative for data collection. In a quantitative instrument, Students' Algebraic Errors Test (SAET) was used to collect quantitative data. The qualitative instrument, algebraic misconceptions interview questionnaire (AMIQ) was used to explore students' misconception based on their answers obtained from the quantitative part. The purpose of interviewing is to enable the respondents to spell out their understanding of the concept (Best and Kahn, 2012). The interview was conducted two weeks after the test, and two (2) students were interviewed from selected school to identify their Misconceptions on the concept. The interview lasted between 20 to 30 minutes. Voice recorder via the smart phone were use and later transcribed. The instrument was validated by the experts and the reliability coefficients of SAET and AMIQ were obtained as 0.82 and 0.86 respectively. Data collected were analyzed using descriptive statistics and thematic content analysis.

Results

The results were presented based on the research questions as follows:

Research Question One

What are the categories of errors made by senior secondary school students in understanding algebraic variables?

Based on the analysis of the students' worksheet, the category of errors identified are presented in Table 1 below.

Table 1: Categories of Errors in Understanding of Algebraic Variables

S/N	Type of Errors	Number of incorrect answers	Percentage (%)
1	Assigning labels, values, or verbs and constants for algebraic variables.	67	18
2	Forming incorrect equations as answers	33	9
3	Lack of understanding the concept variable.	36	10
4	Misinterpreting the product of two variables	194	51

Table 1 shows the majority of errors 194(51%) in understanding of algebraic variables were due to misinterpreting the product of two variables. This indicated that students struggle with understanding the concept of multiplying variables which is a fundamental concept in algebra. The second most common error 67(18%) is assigning labels, values, or verbs and constants for variables, these indicates that students may not fully understand the concept of variable and how to label them, it values and concepts of variables correctly. The lack of understanding the concepts of variables 36(10%) and forming incorrect algebraic equations 33(9%) are also categories of errors.

These errors reveal that students might not have a solid foundation in algebraic concepts and may struggle with applying them to solve problems. As such, the types of categories of errors were shown in the Figure 1 below that provides a sample of students' scripts and their errors in understanding of algebraic variables.

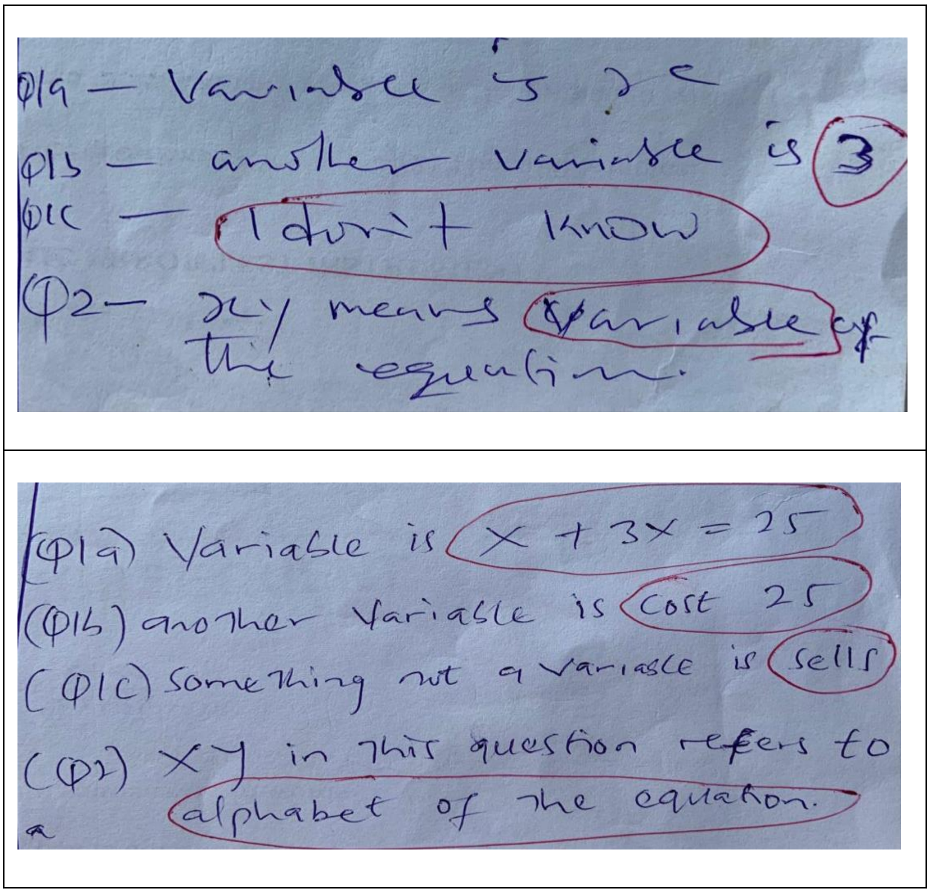

Figure 1: sample: of students' scripts based on question 1 and 2

Figure 1 indicated that students lacked understanding of the concept of algebraic variable by saying "I don't know" and when the participants were asked to named variable and something that is not a variable as well as misinterpreted a variable as a 'label' by saying the variable is 3 or even as a verb such as 'sells. The majority did not interpreted the correct interpretation of the variable as the 'number of a thing'. It was also difficult for students to differentiate between a variable and a non-variable. The students were confused with and viewed variables as constants or vice-versa. This error was noticed when students were asked to name something in the question that was not a variable, the answer such as 'Bello' and 'Musa' were given. In a general sense, these answers may be considered incorrect in the context of the given problem since there was a variable or a given number attached to it. Therefore, these words had meanings in the given context when they are taken together with those variables.

Another identified error in the variable was forming an incorrect equation as answers. Most of student's answers were both in the form of equations $x + 3x = 25$ and $x \times 3x = 25$. There was no meaning attached to these equations, and majority indicate a false relationship between the variables and constants in the problem. More so, it can be observed that, in question 2, majority had difficulties to interpret the product of two variables as two separate variables when combined by a sign. They viewed the product as one variable, constant and letter. Below in the Table 2 shows an average percentage of incorrect responses for students in understanding of algebraic variables.

Table 2 Average Percentage of incorrect Responses for "Understanding of Algebraic Variables"

Question Number	Number of Respondents	Number of Incorrect Answers	Percentage (%)	Average Percentage
1(a)	378	92	24	
1(b)	378	188	50	40
1(c)	378	128	34	
2	378	194	51	

Table 2 presented the percentages and average percentage of incorrect responses for understanding the variables, which revealed that 40% of the respondents out of 378, answered the questions incorrectly in solving problems related to algebraic variables.

Research Question Two

What are the Misconceptions made by Senior Secondary School Students in Understanding Algebraic Variables?

To answer the research question 2, interview was used, where R stands for researcher while S stands for the student who was the interviewee. The sub-section below analysed the interview.

Interview transcript with the S_1

S_1 had a misunderstanding of the concept of a variable. The misconception was seen from answer for question 1(a), and 1(b). The student was asked to name variable in the question, and answered "x". Also, the respondent was asked to give another variable in the question. Then, answered that there is no another variable as seen in Figure 2 below.

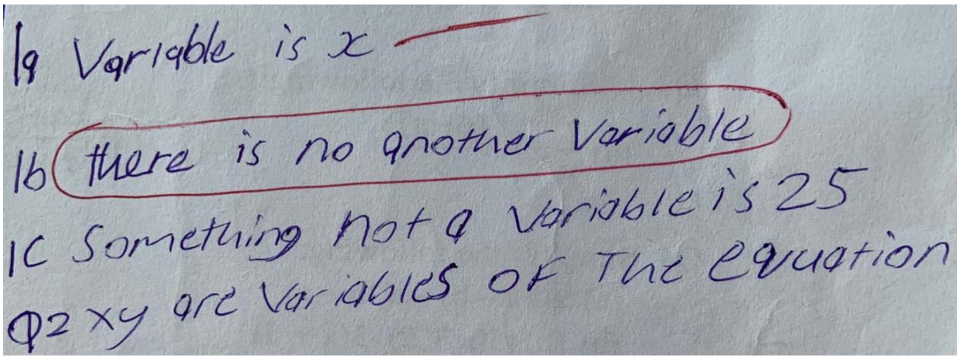
1a Variable is x
1b there is no another Variable
1c Something not a Variable is 25
Q2 xy are Variables of The equation

Figure 2: Extract of S_1 question 1 & 2

Figure 2 extracted from S_1. The answer given in the transcripts was by far contrary to the expected answer. The interview was carried out in order to find out about the S_1 understanding about the variable. The following shows the interview questions and responses with the S_1 and the researcher:

R: What is a variable?
S_1: A variable is a letter in an equation
R: Can a variable take values?
S_1: Yes, it takes values.
R: How many values can it take at a time?
S_1: One value at a time.
R: Why did you say there was no other variable in question 1(b)
S_1: Because the question contained only one letter X, no other letter.
R: When you have 5 mangoes and you give away two, how many remain?
S_1: 3 mangoes
R: Why didn't you say 3 without the mangoes?
S_1: Because it is 3 mangoes
R: How many variables are in 3y + 2y?
S_1: One varible "y"
R: How many variables are in 3x + 2y?
S_1: Two variables x and y.
R: What was your answer to question 2 that says; What does yz mean in yz + 3?
S_1: xy are variables

The S_1 answers show that there is a lack of understanding of the concept of algebraic variables by not perceiving the correct interpretation of the variable as the 'number of a thing'. It was also difficult for S_1 to distinguish between variables and non-variables in terms of the varying and non-varying quantities in the question.

Interview transcript with the S_2

S_2 had a misunderstanding of the concept of a variable. The misconception was seen from answer for question 1(a) which were asked to name variable in the question, and answered x + 3x = 25. Also, misinterpret the product of two variables as a letter, as seen in Figure 3 below.

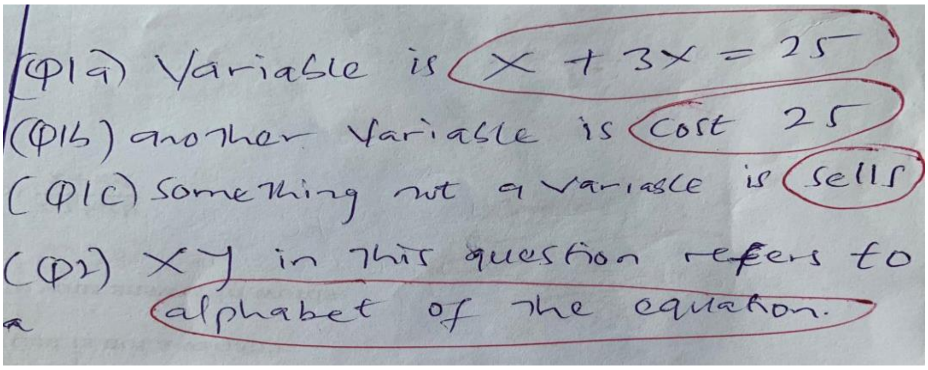

Figure 3: Extract of S_2 question 1 & 2

Figure 3 extracted from S_2 and the transcripts answer was by far contrary to the expected answer. The interview was carried out in order to find out about the S_2 understanding about the variable. The following shows the interview questions and responses with the S_1 and the researcher:

R: What is the Variable as in question 1a ?

S_1: A Variable is x + 3 = 25.

R: How do you get this answer ?

S_1: Bello mango is x, Musa mango is 3x, total mangoes cost 25, therefore x+3x=25.

R: Did you ask to name a variable or to form an equation?

S_1: To name a variable, sir.

R: Why did you form the equation?

S_1: Because the equation contains the variable.

R: Okay, name the variable.

S_1: Variable is x.
R: What is the second variable?
S_1: The second variable cost 25.
R: What was your answer to question 2 , What does yz mean in yz + 3?
S_1: yz are alphabet.
R: I mean in this question.
S_1: Yes sir, yz are alphabet.

The S_2 answers show that there is a lack of understanding of the concept of algebraic variables by not perceiving the correct interpretation of the variable as the ‘number of a thing due to forming a wrong equation as an answer. It was also difficult for S_1 to interpret the product of two variables as two separate variables and viewed it as a letter.

Summary of Major Findings.

The following are the summary of the major findings of the study.

1. The findings indicated that forming of incorrect algebraic equations as variables.

2. Also, the findings revealed that, majority of the students had difficulties for understanding the concept of algebraic variables.

3. The misinterpretation of algebraic variables were identify by many students as a difficulty to perceive the product of two variables as two separate variables.

4. Most of the errors committed by students are not ordinary errors but rather are misconceptions.

Discussion

The findings of this study are discussed based on the four categories of errors related to the understanding of algebraic variables. These included forming of incorrect algebraic equations as variables, lack of understanding the concept of variables, misinterpretation of algebraic variables, and misconceptions due to errors committed by the students rather than the errors per se.

The category of errors that the students form in relation to incorrect equations as variables. These incorrect equations as variables was similarly found as the difficulties in learning algebra (Pramesti & Retnawati, 2019). However, lack of understanding of basic concepts is a possible reason for this misconception. It is noted in this study that students' responses to the interview questions were contradictory to each other and continuously seemed to be confused and could not provide good explanations.

Likewise, some students had difficulty for understanding the concept of variable when they were asked to name a variable in the problem given. This difficulty for understanding the concept coincided with the errors given in the literature that, the error has a link with levels of cognitive development (Egodawatte, 2011; Tekin-Sitrava, 2017). This means that if a student has slowed mental development, then that students might make more errors and misconceptions than a student who has mental development (Fitria & Susanto, 2023).

More so, misinterpreting the product of two variables was also identified by many students. The students who made such errors had difficulty to perceive the product of two variables as two separate variables combined together by a sign. This is in line with research in which the students misinterpreted algebraic variables, though they sometimes succeeded in determining variable values from algebraic expressions (Fitria & Susanto, 2023). Also, the findings coincide with the study in which students misinterpreted a variable as a label or as a thing or a verb as they failed to perceive the variable as the number of a thing (Ndemo & Ndemo, 2018).

Conclusion

It can be concluded that Students committed a lot of errors in relation to algebraic variables. However, the committed errors by the students are not errors per se, rather a misconception. Hence, Mathematics teachers should be made to seek a possible way of overcoming the identified errors and misconceptions. Also, it could be concluded that understanding students' concept related to the algebraic variable, will provide opportunities for educators to learn students’ needs in order to improve their understanding of algebraic variables.

Recommendations

Based on the findings of the study, the following recommendations were made:

1. It is recommended that teachers should involve their students in activities that will aid their learning and algebraic concepts. These will make students to be conscious of the various algebraic concepts and improve to overcome their errors and misconceptions.

2. It is recommended that the curriculum implementers who prepare teachers for pre-service and in-service training should lay emphasis on best teaching method so as for the students to overcome all these errors and misconceptions.

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